QUANTUM LOGIC IS ALIVE
$\wedge$ (IT IS TRUE $\vee$ IT IS FALSE)

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1. The Reports of Its Demise Are Premature

Is the quantum-logic interpretation dead? Its near total absence from current discussions about the interpretation of quantum theory suggests so. While mathematical work on quantum logic continues largely unabated, interest in the quantum-logic interpretation (as found in (Hooker, 1979), for example) seems to be almost nil, at least in Anglo-American philosophy of physics.

This paper has the immodest purpose of changing that fact. I shall argue that while the quantum-logic interpretation faces challenges, it remains a live option. The usual objections either miss the mark, or admit a reasonable answer, or fail to decide the issue conclusively.

The next section describes a quantum-logic interpretation, one that seems defensible (and henceforth called 'QL'). Nothing that is said there should be taken to illuminate or to be illuminated by other proposed quantum-logic interpretations, notably those of Putnam (1969) and Finkelstein (1962; 1969). I lack the space to say how QL is related to those interpretations. (It is, of course, not wholly distinct from them. Other important influences are (Friedman and Putnam, 1978) and (Demopoulos, 1976).) Section 3 answers the standard objections to quantum-logic interpretations, as applied to QL. Apart from historical contingencies, these objections seem to motivate the current disdain for quantum-logic interpretations.

This paper owes a great deal to numerous conversations that I have had with Michael Friedman. I am also indebted to Matt Frank for some useful correspondence. (Any errors are due to them.)$^{\perp}$.

2. A Quantum-Logic Interpretation

Quantum Logic. Suppose that quantum-mechanical propositions take the form 'observable F has a value in (Borel) set Δ.' In the standard Hilbert-space formulation of quantum theory, such propositions are represented by (closed) subspaces of a Hilbert space, H. (I follow the usual practice of denoting subspaces, projections, and propositions by the same symbols, and move freely amongst these nonetheless distinct concepts.) The set of all such subspaces forms an ortholattice, $L(H)$, with $p \leq q$ defined by 'p is a subspace of q'. In quantum logic, the logical operations of 'and', 'or', and 'not' are modeled respectively by the operations of meet ($\wedge$, infimum), join ($\vee$, supremum) and orthocomplement ($^{\perp}$) on $L(H)$. Given these operators (and some symbols, such as parentheses), one defines the formal sentences of quantum logic in the obvious way.

Any proposition equal to H is logically true. One proposition, p, semantically entails another, q, just in case $p \leq q$. Quantum logic is non-distributive whenever the propositions involved are not members of a common Boolean sublattice (i.e., whenever they are 'incompatible'). Let p, q, and r be incompatible. Then $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ may denote distinct subspaces, and so are not semantically equivalent. (Consider, e.g., any three distinct coplanar rays.)

If H is infinite-dimensional, one must give a more careful account of quantum logic. (If one works within the algebraic formulation of quantum theory, as I would advocate, then one must be more careful still.) I lack the space to do so here, and so I shall generally assume that H is finite-dimensional, though almost everything I say does not depend on this assumption.

The Fundamental Claim of QL. QL claims that quantum logic is the 'true' logic. It plays the role traditionally played by logic, the normative role of determining right-reasoning. Hence the distributive law is wrong. It is not wrong 'for quantum systems' or 'in the context of physical theories' or anything of the sort. It is just wrong, in the same way that '(p or q) implies p' is wrong. It is a logical mistake, and any argument that relies on distributivity is not logically valid (unless, of course, distributivity has been established on other grounds). Anything else claimed by QL is in the service of supplementing this claim. I shall consider some such subsidiary parts of QL below.

The Motivation. What motivation is there for QL? I begin with an obvious objection, often mentioned in the literature: the fact that one can *define* a logical connective, or indeed a formal language (synatx and semantics), in no way implies that the connective, or the language, has anything to do with 'logic' in the proper sense, that is, reasoning. In particular, it in no way implies that the connectives of the language have any significance beyond their formal definition.

The objection, as I say, is often leveled against quantum-logic interpretations, but the question it raises will in fact lead us to a motivation for QL. We may ask: what is the significance—and especially the *empirical* significance—of the logical connectives? (I mean *the* logical connectives, the ones we use to reason, to construct arguments, and so on.) Suppose that p and q are empirically significant propositions. What is the empirical significance of the propositon 'p and q', and so on?

To see how we might answer this question in a non-circular way, it might help to consider the converse of the usual connection between quantificational concepts and the concepts of 'and' and 'or'. In particular, let us take 'p or q or r or ... ' to mean that some of p, q, r, ... is (or are) true, and 'p and q and r and ... ' to mean that all of p, q, r, ... are true. Hence 'some' and 'all' become the fundamental concepts, while 'and' and 'or' may be defined in terms of them. To establish the empirical significance of 'p and q', then, we ask: what is the empirical significance of the claim 'all of $\{p,q\}$', and so on. (Do not read too much into this move. It is made here for heuristic purposes only. I am *not* making a deep claim in the philosophy of logic.)

Such a question cannot be answered outside the context of a physical theory, and therefore QL consults quantum theory. (Indeed, under the independently reasonable assumption that the empirical world obeys the laws of logic, it would behoove us in any case to study the empirical world in order to learn something about logic. Of course, some philosophers think that they know all about logic prior to studying the world, but the world has a nasty way of surprising those people.) Given propositions (subspaces) p and

q, with which proposition does quantum theory associate the proposition 'some of $\{p, q\}$' (henceforth, 'some p, q')? The question is tricky, because if p and q are incompatible, then we cannot test the truth of 'some p, q' in the obvious way, namely, by simultaneously measuring them. However, we *can* discover whether *neither* of them is true: if r is true and orthogonal to both p and q, then neither p nor q is true (because then they both have probability zero). Hence QL proposes that the sentence 'some p, q' has the following empirical significance:

- If p is true then 'some p, q' is true, and similarly for q.
- If 'some p, q' is true, then not both p and q have probability zero.

The first condition entails that p and q are subspaces of 'some p, q', and the second entails that 'some p, q' cannot be larger than $p \vee q$ (for if it were, then it could be true while both p and q have probability zero). Exactly one proposition in $L(H)$ meets these conditions, namely, the (closed) span of p and q. Similar considerations suggest the logical character of the lattice-theoretic meet. If we require that the negation satisfy De Morgan's laws, then we can similarly establish the logical character of the orthocomplement. (Work by Dunn (1993) suggests a number of ways to establish the logical character of the orthocomplement, including the way cited here.)

In fact, unless one is willing to deny the empirical significance of 'some p, q', it is hard to resist this conclusion—$p \vee q$ is the only plausible candidate in quantum theory for this proposition. Hence most interpretations[1] *do* deny the empirical significance of 'some p, q' for most p and q, by picking some *preferred subset*, S, of $L(H)$ as the set of meaningful propositions (perhaps at a given time, or in a given world, or in a given context). These interpretations *continue* to represent logical operations in terms of the lattice-theoretic operations on $L(H)$, but those operations are restricted to S, and S is chosen so that the resulting logic is distributive (or at least effectively so[2]).

[1] I mean to include at least Bohm's theory, the modal interpretations, the many-somethings interpretations, the spontaneous collapse interpretations, and the Copenhagen interpretations.

[2] See, e.g., (Dickson, 1996) for a discussion of this point in relation to modal interpretations.

QL, on the other hand, asserts the simultaneous well-definedness of all quantum propositions. Hence, while $\bigwedge_{p\in L(H)}(p \vee p^{\perp}) = H$, a preferred-subset interpretation denies that it expresses a meaningful proposition, because many of its components do not express meaningful propositions in the first place (or at least not propositions that can be simultaneously meaningful).

Conditionals. Hardegree (1979) has shown that the Sasaki hook, $\rightarrow$, defined by $(p \rightarrow q) \equiv (p^{\perp} \vee (p \wedge q))$, is the unique connective on an ortholattice satisfying modus ponens and the following two conditions:

- $p \wedge (p^{\perp} \vee q) \leq r$ implies $q \leq (p \rightarrow r)$
- $p \wedge q \leq (p \rightarrow q)$.

We may note, as well, that $p \leq q$ implies and is implied by $p \rightarrow q = H$. I.e., $p \rightarrow q$ is logically true if and only if $p \leq q$. Moreover, if $p \rightarrow q = x < H$ then $x|p \leq x|q$, where $x|y$ is the projection of y onto x, i.e., $x|y = x \wedge (x^{\perp} \vee y)$.[3] In other words, if we are 'given' that $p \rightarrow q$, then upon conditioning in light of this knowledge, we find again that p, conditioned on $p \rightarrow q$, entails q, conditioned on $p \rightarrow q$. (Indeed, it entails q.)

Hardegree has provided a useful characterization of the Sasaki hook as a kind of 'counterfactual' conditional. Suppose, for example, that r is incompatible with p and q. Still it may be possible to assert r together with $p \rightarrow q$, where we read $r \wedge (p \rightarrow q)$ as 'r and, if we could assert p, then we would be able to assert $p \wedge q$ (and therefore q)'. Such a conditional is all we should expect from quantum logic. Indeed, it is exactly what we should expect given the existence of incompatible propositions.

Dynamics. Dynamics is given in the usual quantum-mechanical way by unitary time-evolution operators: if p is true at time t_0 and $U_{t_0,t}$ is the time-evolution operator over the interval $[t_0, t]$ then $U_{t_0} p U_{t_0}^{-1}$ is true at time t.

[3]Proof. $x|p$ is

$$[p^{\perp} \vee (p \wedge q)] \wedge [(p^{\perp} \vee (p \wedge q))^{\perp} \vee p].$$

Use De Morgan's law and the fact that $(p \wedge z) \vee p = p$ for any z to get

$$x|p = [p^{\perp} \vee (p \wedge q)] \wedge p.$$

p and $(p \wedge q)$ are compatible, and so we can use distributivity to get $x|p = p \wedge q$. On the other hand, $x|q$ is

$$[p^{\perp} \vee (p \wedge q)] \wedge [(p^{\perp} \vee (p \wedge q))^{\perp} \vee q].$$

But $[p^{\perp} \vee (p \wedge q)] \geq (p \wedge q)$ and $[(p^{\perp} \vee (p \wedge q))^{\perp} \vee q] \geq q \geq (p \wedge q)$ and so $x|p \leq x|q$.

However, we must take some care in saying clearly what we mean by the previous statement, which is a conditional. To discuss it, we need to extend the formalism to allow propositions of mixed times, e.g., 'p is true at t_0 and q is true at t'.

Let us begin, for simplicity, with the assumption that time is discrete. Then we can associate each time with an integer, and with each integer, a Hilbert space, H_n, so that $L(H_n)$ represents the properties of the system at the time n. Propositions of mixed times are then elements of $L(H)$, with $H = \bigotimes_n H_n$, the (countably infinite) tensor-product of Hilbert spaces H_n.[4] For example, the proposition

$$I_1 \otimes \cdots \otimes I_{m-1} \otimes p \otimes I_{m+1} \otimes \cdots \otimes I_{n-1} \otimes q \otimes I_{n+1} \otimes \cdots$$

is the proposition 'p occurred at t_m then q occurred at t_n', and so on. (While an adequate treatment of infinite tensor products is a subtle business, many of the general features of dynamics as described here would remain.) Finally, dropping the simplification of countably many times, we would have to work with a tensor-product of *continuously* many Hilbert spaces, one for each time. This mathematical object has been defined (Isham et al., 1998, and references therein), but I shall not discuss it here.

Simplifying even further to the case of just two times, when can we say that the proposition $p \otimes I$ (i.e., 'p occurs at the earlier time') 'dynamically implies' (evolves to make true) the proposition $I \otimes q$, written $p \hookrightarrow q$? The logic of these 'mixed-time' propositions must be derived from the semantics (given by quantum theory), just as in the simpler case of single-time propositions.

The conditional $\hookrightarrow$ is a strict conditional, and so not expressible as an element of $L(H)$. Instead, we specify the set, S, of *dynamically possible* evolutions by

$$p_1 \otimes p_2 \in S \quad \text{iff} \quad p_2 U_{(1,2)} p_1 U^{-1}_{(1,2)} p_2 \neq 0, \tag{1}$$

where $U_{(1,2)}$ is the evolution operator over the inverval $[t_1, t_2]$. (The generalization to a countable infinity of times is clear. The generalization to

[4] The infinite tensor product was first defined for Hilbert spaces by von Neumann (1939). The proposal to treat mixed-time propositions in this way is central to the so-called 'histories' approach to quantum theory, which has ocassionally flirted with quantum logic—see, e.g., (Omnès, 1994) and (Isham, 1994).

continuous time is considerably more involved.) Then $p \hookrightarrow q$ if and only if $p \otimes q^{\perp} \notin S$.

From this semantics, which is evidentally the correct semantics for $\hookrightarrow$ regardless of one's views about quantum logic, one can derive the logical properties of $\hookrightarrow$, but there is no need to do so here. The point is that $\hookrightarrow$ can be defined as a kind of strict conditional, and its truth is indeed some kind of universal generalization over all possible histories (elements of S).[5]

Probabilities. QL adopts an ignorance interpretation of probabilities in quantum theory. Gleason's theorem then immediately delivers the quantum-theoretic probabilities. Suppose, for example, that you come to know that p, a maximal (one-dimensional) proposition. Then you should assign p probability 1, and in that case Gleason's theorem fixes the probability that you assign to other propositions, because it entails that there is exactly one probability measure that assigns probability 1 to p. When p is not maximal, the situation is only slightly more complex.[6]

The Kochen-Specker Theorem. The Kochen-Specker theorem (the theorem that there is no homomorphism from $L(H)$ to the Boolean algebra Z_2) is usually taken to rule out the idea that every (empirical) proposition (as defined at the beginning of this section) has a truth-value (henceforth, 'naïve realism'). From the point of view of QL, the Kochen-Specker theorem shows, instead, that distributivity is false, because the existence of such a homomorphism from an ortholattice to Z_2 is equivalent to distributivity of the ortholattice (Demopoulos, 1976; Ptàk and Pulmanova, 1991, e.g.).

3. Objections

Those familiar with quantum-logic interpretations will have already become frustrated that I have not addressed a number of serious objections. I turn, in this section, to that task. To start, I should dismiss a number of objections that are irrelevant to QL (but possibly relevant to other quantum-logic interpretations).

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[5]Note that the notion of a 'history' used here is considerably more general than that used in so-called 'consistent histories' interpretations of quantum theory, where histories must be such as to render classical probability theory valid. No such requirement is made here.

[6]See Dickson (1998) for a discussion and references, including a reference to (Friedman and Putnam, 1978).

First, QL does not assert a 'logical split' of any sort between the classical and the quantum realms. Quantum logic is *the* true logic. QL need not, therefore, give an account of the distinction between 'the realm in which quantum logic is true' and 'the realm in which classical logic is true'.

Second, QL does not propose to analyze the two-slit experiment as Putnam (1969) does. I mention Putnam explicitly here because his analysis has been the object of numerous criticisms, perhaps most notably by Gibbons (1987). Putnam's analysis uses classical probability theory, and proceeds roughly as follows. Let p_1 [p_2] represent the proposition 'the particle traversed the first [second] slit' and let r represent the proposition 'the particle landed in the region r on the screen'. Putnam then suggests that the two-slit experiment appears 'mysterious' only if we suppose that

$$\begin{aligned}&\Pr(r|p_1 \vee p_2)/\Pr(p_1 \vee p_2)\\&\qquad = \Pr(r|p_1)/\Pr(p_1 \vee p_2) + \Pr(r|p_2)/\Pr(p_1 \vee p_2),\end{aligned} \tag{2}$$

which relies on distributivity, and so is generally invalid in quantum logic. There are three problems with this account. First, it uses the classical definition of conditional probabilities (because Putnam accepts that the left-hand side of (2) is correct); second, r, p_1, and p_2 are compatible; and third, the account fails to take notice of the essentially dynamical aspect of the two-slit experiment (and of interference experiments in general). Gibbons' critique relies on these points.

Third, QL is not operationalistic. Some advocates of quantum-logic interpretations (e.g., Finkelstein, 1969) have suggested that the logical connectives can be defined in more or less operational terms, so that there is a *direct* connection between statements in a formal language such as 'p and q' and experimental procedures for testing such statements. However, QL takes seriously the idea that its proposal for the empirical significance of 'and' and 'or' is just that—a proposal. Moreover, QL does not even claim that there is an adequate operational definition of 'some' and 'all'. Rather, the claim is that, in the context of quantum theory, there are some reasonably clear empirical conditions that must be met if we can say that the proposition 'some p, q' is true, assuming that 'some p, q' is a meaningful propositon in the first place (i.e., assuming 'naïve realism').

A related point is that some proponents of quantum-logic interpretations have argued that there is a set of core formal properties that 'and' and 'or' must meet. For example, 'p and q' must imply p. Having defined these 'core' properties, and insisted that they consitute the 'meaning' of the logical connectives, they then demonstrate (very quickly, because the demonstrations are trivial) that the quantum-logical connectives are the only connectives in $L(H)$ that meet these formal conditions. The obvious objection to this procedure—voiced, e.g., by Gibbons (1987)—is that the conditions are chosen precisely to give the desired result, while those conditions not satisfied by the quantum-logical connectives are arbitrarily left out. QL does not argue from formal conditions that must be satisfied by 'and' and 'or'. Rather, according to QL, it is a matter of empirical and theoretical investigation to develop a physical theory from which we can discover the formal properties of 'and' and 'or'. If we accept incompatibility (hard to deny, given quantum theory) and naïve realism, then QL's proposal for the empirical significance of 'and' and 'or' is at least not unreasonable, and perhaps inevitable in the context of quantum theory.

Fourth, one may, of course, deny naïve realism. My aim, however, does not encompass answering such objections. Instead, I aim only to answer objections that call into question the adequacy of QL as an interpretation of quantum theory. I have no desire to advocate QL, but only to make clear its status as a potentially viable interpretation. Let us now consider objections to this conclusion.

Classical logic is necessarily true. The objection, more precisely, is that the 'meaning' of the logical connectives is given independently of empirical considerations. Dummett (1976) voiced this idea in objection to Putnam:

> The idea that the meanings of the sentential operators are given by truth-tables, and that, being so given, the distributive law cannot but hold for them, never gets a glance.

On this 'truth-table' account, the meanings of, and therefore the laws obeyed by, the connectives are established by appeal to truth-tables. Any apparent violation of logic by a theoretical description of the world therefore requires us to rethink our theoretical description of the world. Logic is not open to revision. Proponents of the truth-table account *must* find a description of the world (and a way of connecting logical laws with that description) that obeys the laws of logic. This search is easy in the case of classical mechanics. It is somewhat more difficult in the case of quantum theory.

That fact alone motivates some philosophers—notably, proponents of quantum-logic interpretations—to consider alternatives to the truth-table view. What, exactly, are they giving up? The truth-table view seems to come from some principle such as the following: if each of two sentences is true under the same conditions, then the sentences mean the same. But is this principle compelling? Consider the following two sentences: $p \wedge q$, and

$$\Big\{\big(p \wedge [(p \rightarrow p) \vee q]\big) \wedge [q \rightarrow (p \wedge q)]\Big\} \wedge \Big\{[(q \vee \neg p) \vee \neg q] \wedge q\Big\}.$$

They are (classically) true under the same conditions. Do they mean the same? It is not obvious (to me) that they do. But if not, then 'truth under the same conditions' is not sufficient for 'sameness of meaning', and the account of the meaning of the sentential connectives in terms of truth tables loses its initial appeal. In other words, those who give up the truth-table view are not giving up anything that is obviously true.

More important, the truth-table view makes it unclear why the laws of logic determine right-reasoning. Of course, any argument that self-avowedly employs the connectives defined in terms of truth-tables must obey the rules derived from those truth-tables (e.g., distributivity); but why suppose that in general our reasoning does employ those connectives, so defined? What evidence could we have for this claim? The only evidence that I can imagine—apart from personal testimony—is that known cases of good reasoning obey the laws derived from truth-tables.

Let us pause, here, to consider what personal testimony would be required. Good reasoners would say things such as 'whenever $p \wedge q$ is true, both p and q are true' or '$p \vee q$ is true just in case either p is true or q is true', and so on. Such statements get us nowhere, assuming (as we are) that there is no distinction between the logic of the object-language and the logic of the meta-language. The only testimony about the *meanings* of the logical connectives that would decide the issue would be the testimony of those who, like Dummett, explicitly appeal to truth-tables to establish the meanings of logical connectives.

Unless we are prepared to accept that testimony as relevant evidence, it seems that we are in the following situation: we have a collection of recognized, paradigmatic, examples of good reasoning, and the question then becomes how best to describe that collection formally. The validity of our logical inferences, under this conception, is not a function of the formal model. Rather, the formal model is an attempt to capture the notion of valid reasoning (perhaps, for example, so that it can be extended to areas where we are less sure of how to proceed, or we are less sure of what counts as 'good reasoning').

What counts as an example of good reasoning, and how do we come to know what those examples are? Do we 'just know' one when we see it? No. We must have some way to assign intersubjectively available truth-values to sentences involving logical connectives. In other words, we need some way to determine whether a given form of logical inference is 'successful'. Given the principle that the world (or, a correct theoretical description of the world) does not 'disobey' the correct rules of reasoning, we may hope to determine which inferences are correct by determining how to relate propositions involving logical connectives to empirical facts. QL proposes one such way.

There is no real evidence for QL. However, there is an objection to the very idea that we *do* thereby arrive at the truth of quantum logic. The point is this: there is good reason (given the ubiquity of decoherence) to believe that the propositions applicable to the macroscopic world are, or are effectively, compatible. (By 'effectively compatible' I mean that the probability of finding interference effects, or violations of distributivity, is extremely small.) But then any attempt to verify (or refute) a putative law of logic by means of an empirical test will always turn out in favor of classical logic. Indeed, we need not invoke decoherence: because we can never perform an empirical test involving incompatible propositions, we can never find even an apparent violation of classical logic.

However, QL never proposed that laws of logic are tested empirically. Rather, it proposes that physical theories contain information about the logical laws obeyed by the world. To extract this information, QL proposes that the proposition 'some p, q' is always meaningful, and then considers what empirical evidence could bear on the truth or falsity of this proposition when p and q are not simultaneously measurable. The answer came in the form of two conditions (see the previous section), and quantum theory is rich enough to tell us something about how to meet those conditions. QL does not propose that logic can be 'read off of' empirical facts.

QL is not explanatory. Proponents of quantum-logic interpretations—notably, Putnam—have argued that the usual mysteries of quantum theory are explained away by quantum-logic interpretations. (Putnam's analysis of the two-slit experiment was an attempt to show that it is not mysterious.) One might be tempted to object that QL provides no explanations of quantum mysteries, but only expresses them in a formal language.

There are two explanatory tasks that one might have in mind, here. The first is the task of explaining why various phenomena as described by quantum theory seem so mysterious to us, that is, why they are difficult for us to understand. QL addresses this point by proposing an account of why logic *seems* classical to us. I shall discuss this point below. The second is the task of explaining why those phenomena are as they are (e.g., why there are interference phenomena). There are two questions, here.

1. *Why are the laws of logic the way they are?* QL has no good answer to this question. The best one can do, I think, is what Aristotle does in the *Metaphysics* in his attempt to argue for the principle of non-contradiction. "It is a lack of education not to know of what one should seek a demonstration" (1006a7-8, my translation), says Aristotle, and so he seeks not a demonstration, but an argument that those who deny the principle will be stuck saying (or believing) false things about the world. Of course, to make such an argument, one must have some antecedent view about the way the world is. QL takes its antecedent view about the way the world is from quantum theory, plus 'naïve realism'. The argument, then, is that if one denies a law of quantum logic, one will end up saying false things about the world, by these lights. Granted, this argument provides no explanation of the truth of those laws.

2. *Given the laws of logic, why are the phenomena the way they are?* QL does not argue for the truth of quantum logic by appeal to its explanatory power. Nonetheless, having adopted quantum logic as the true logic, one can indeed get very far towards understanding the structure of quantum theory.

The main results are due to Piron (for which, see (Ptàk and Pulmanova, 1991)) and Solèr (1995). They begin by defining an abstract quantum logic as a system of propositions with connectives obeying certain conditions (those obeyed by the quantum-logical connectives, as I defined them above). Piron showed that such systems constitute complete lattices, where the partial order is semantic implication, and that these lattices are representable as lattices of subspaces of a vector space. Solèr more or less completed the investigation by showing that in fact the vector space in question is a Hilbert space over the real numbers, complex numbers, or quaternions. In other words, there is a close connection between the truth of quantum logic and the use of Hilbert spaces in quantum theory. More than this connection is unreasonable to expect. After all, QL certainly does not claim that logic alone is sufficient to determine the form of a physical theory.

QL does not account for the success of classical logic. I have emphasized that QL asserts the universal truth of quantum logic. Why, then, does classical logic work so well? The quick answer is that until the development of quantum theory, we only ever reasoned about compatible sets of propositions, or in any case sets that are 'effectively' compatible (see above). In particular, as long as physics concerned itself with more or less macroscopic phenomena, there could be no indication that the world obeys a logic other than classical logic, for reasons that I have already mentioned.

It is wrong, however, to say that we have been 'duped' into thinking that classical logic is valid. It *is* valid, when restricted to certain domains. So while the forms of inference that are used in classical logic are not universally valid, quantum theory (with quantum logic) entails their validity in restricted domains. It is crucial to realize that QL itself entails that our reasoning about classical objects has all along been acceptable, just not for quite the reason that we thought.

QL is expressed in terms of classical logic. Just as classical logic has worked well for reasoning about the macroscopic world, so also it seems to have worked quite well in mathematics. In particular, the mathematics of Hilbert spaces, lattices, and so on, has been developed with classical logic. But how can QL account for this fact?

Answering this question requires the completion of a difficult and presumably long program of 'recovering' mathematics from quantum logic (or at least any mathematics about which we feel reasonably confident, and especially, of course, those parts of mathematics that are used in science). I cannot say whether this program could be carried out convincingly, but some small parts of it have been successful. For example, Dunn (1980) has shown that arithmetic done within a quantum-logical framework contains all of the same theorems as classical first-order Peano arithmetic. In particular, distributivity (of 'and' over 'or', not multiplication over addition) is really a theorem of arithmetic, rather than of logic—but it is a *theorem*. Classical mathematicians have not been 'mistaken' to use distributivity in arithmetic. They were just not quite right about the *reason* that distributivity is a theorem.

The situation for higher-order theories is more complex—in particular, the quantum logician must make some hard decisions about extensionality—so let me just say that the work by Dunn (and others) suggests that quantum logic might provide an adequate foundation for our mathematical practice. However, this program might also fail, and if it does, then QL will indeed be implausible.

QL is not 'real realism'. The objection here is, essentially, that quantum logic does not solve the measurement problem (though we can extend the point to call into question QL's claim that 'every proposition has a truth-value'). The problem is that quantum theory apparently fails to account for the fact that at the end of the measurement, the measuring device indicates p_1 or p_2, or, … , where the p_n are the possible results. The argument is that the 'or' in that previous sentence is the classical 'or'. That is, we do not say that the device indicates "p_1 quantum-or p_2, quantum-or, … ," but that it indicates "p_1 classical-or p_2, classical-or, … ", and yet QL establishes only the former, which, by the way, was exactly the source of the problem in the first place. QL has three replies.

1. If we are primarily concerned with the measurement problem, then the objection loses its force once we realize that the possible indicator-states of an apparatus will be compatible. In that case, as I have already discussed, the logic of statements about those indicator-states is the same whether we

use classical or quantum reasoning. The only way to resurrect the objection in this case is to insist that although the logic of statements about these indicator-states is exactly the same in both cases, still we 'meant' to be saying that this logic is a consequence of the use of classical logic, not of quantum logic. I find it difficult to imagine, however, that the intuitions behind our concern over the measurement problem are so logically specific. Rather, it seems that we mean merely to assert that 'some result is indicated', and it is up to the philosopher and logician to figure out how to represent that fact in a formal system. QL proposes that it is represented quantum-logically.

2. To accept reply 1 is to give in almost all the way, but one might still hold out and say that nonetheless, the generic 'realism' embodied in the logical truth of $\bigwedge_{p \in L(H)}(p \vee p^{\perp})$ is no realism at all, because realism somehow includes the idea that classical logic is true, and in particular that distributivity holds. Hence Dummett, for example, says that 'to say that every statement was either true or not true, when 'or' was used in such a [quantum-logical] way, would not be an expression of realism,' and that 'the "or" that was used [in the characterization of realism] must be taken as one for which the distributive law holds.'

Why? I am skeptical that the doctrine of realism—or in any case, whatever of it seems worth saving—is so logically precise a doctrine. Consider two incompatible propositions, p and q. Suppose somebody asserts 'some p, q'. Do we primarily understand this assertion logically? Do we think that we know, without considering a physical theory, what follows from it? Or do we instead think that 'some p, q' asserts something about the world, something that a physical theory can tell us how to test, and something whose consequences are given to us by a physical theory? In other words, QL assumes realism—the well-definedness of propositions such as 'some p, q' for all p and q—and consults quantum theory to discover the logical consequences of such assertions. This consultation happens in the way that I described in the previous section.

4. The Status of QL

QL faces some stiff challenges. The details of the logic of dynamical (mixed-time) propositions has yet to be specified for the realistic case of

$H =$ the tensor product of one Hilbert space for each time. The program of quantum mathematics has hardly gotten off the ground. And no doubt there are philosophical puzzles and objections yet to arise. Nonetheless, I hope to have shown that QL is respectable, or at least that it is not obviously unrespectable. That conclusion may seem somewhat less than grandiose, but given the current state of the quantum-logic interpretation, I can live with it.

References

Demopoulos, W. (1976). The Possibility Structures of Physical Systems. In Harper, W. L. and Hooker, C. A., editors, *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science*, pages 55–80. D. Reidel, Dordrecht, Holland.

Dickson, M. (1996). Logical Foundations for Modal Interpretations. In *PSA 1996*, volume 1, pages 322–329, East Lansing, MI. Philosophy of Science Association.

Dickson, M. (1998). *Quantum Chance and Nonlocality.* Cambridge University Press, Cambridge.

Dummett, M. (1976). Is Logic Empirical? In Lewis, H., editor, *Contemporary British Philosophy*, pages 45–68. Allen and Unwin, London.

Dunn, M. (1980). Quantum Mathematics. In *PSA 1980*, volume 2, pages 512–531, East Lansing, MI. Philosophy of Science Association.

Dunn, M. (1993). Star and Perp: Two Treatments of Negation. In Tomberlin, J., editor, *Philosophical Perspectives: Language and Logic*, volume 7. Ridgeview Press, Atascadero.

Finkelstein, D. (1962). The Logic of Quantum Physics. *Transactions of the New York Academy of Sciences*, 25(2):621–637.

Finkelstein, D. (1969). Matter, Space, and Logic. In Wartofsky, M. and Cohen, R., editors, *Boston Studies in the Philosophy of Science*, volume 5, pages 199–215. Humanities Press, New York.

Friedman, M. and Putnam, H. (1978). Quantum Logic, Conditional Probability, and Iinterference. *Dialectica*, 32:305–315.

Gibbons, P. (1987). *Particles and Paradoxes: The Limits of Quantum Logic.* Cambridge University Press, Cambridge.

Hardegree, G. (1979). The Conditional in Abstract and Concrete Quantum Logic. In *Hooker (1979)*, volume 2, pages 49–108. D. Reidel, Dordrecht.

Hooker, C. A., editor (1975,1979). *The Logico-Algebraic Approach to Quantum Mechanics*, volume 1,2. D. Reidel, Dordrecht.

Isham, C. (1994). Quantum Logic and the Histories Approach to Quantnum Theory. *Journal of Mathematical Physics*, 35:2157–2185.

Isham, C., Linden, N., Savvidou, K., and Schreckenberg, S. (1998). Continuous Time and Consistent Histories. *Journal of Mathematical Physics*, 39:1818–1834.

Omnès, R. (1994). *The Interpretation of Quantum Mechanics*. Princeton University Press, Princeton, NJ.

Ptàk, P. and Pulmanova, S. (1991). *Orthomodular Structures as Quantum Logics*. Kluwer, Dordrecht.

Putnam, H. (1969). Is Logic Empirical? In Cohen, R. and Wartofsky, M., editors, *Boston Studies in the Philosophy of Science*, volume 5, pages 181–206. D Reidel, Dordrecht.

Solèr, M. P. (1995). Characterization of Hilbert Spaces by Orthomodular Spaces. *Cmmunications in Algebra*, 23:219–243.

von Neumann, J. (1939). On Infinite Direct Products. *Compositio Math*, 6:1–77.